Mathematics for Microeconomics

Part I: Derivatives

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Selected Rules of Differentiation

Assume b, c, and m are constants:

1. If $f(x) = c$, then $f^{'}(x) = 0$.
2. If $f(x) = mx + b$, then $f^{'}(x) = m$.
3. If $f(x) = x^n$, then $f^{'}(x) = nx^{n-1}$.
4. If $g(x) = cf(x)$, then $g^{'}(x) = cf^{'}(x)$.
5. If $h(x) = g(x) + f(x)$, then $h^{'}(x) = g^{'}(x) + f^{'}(x)$. If $h(x) = g(x) - f(x)$, then $h^{'}(x) = g^{'}(x) - f^{'}(x)$.
6. If $h(x) = \sum_{i=1}^{n} g_i(x)$, then $h^{'}(x) = \sum_{i=1}^{n} g_i^{'}(x)$.
7. If $h(x) = f(x)g(x)$, then $h^{'}(x) = f^{'}(x)g(x) + f(x)g^{'}(x)$.
8. If $h(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$, then $h^{'}(x) = \frac{f^{'}(x)g(x) - f(x)g^{'}(x)}{[g(x)]^2}$.
9. If $y = f(u)$ and $u = g(x)$ so that $y = f(g(x)) = h(x)$, then $h^{'}(x) = f^{'}(u)g^{'}(x)$ or $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.
10. If $y = \ln x$, then $dy/dx = 1/x$.

Partial Derivatives

Suppose we have a production function defined as $y = f(x_1, x_2, ..., x_n)$, where x's could be labor, capital, and land. For a function of several variables, the idea of the derivative is not well-defined. Thus, we use partial derivatives (or directional slopes) to understand

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the impact of a specific x while holding all the other variables constant. The partial derivative of y with respect to x_1 is denoted as:

$$\frac{\partial y}{\partial x_1} \text{ or } \frac{\partial f}{\partial x_1}.$$

For example, if $y = f(x_1, x_2) = ax_1^2 + bx_1x_2 + cx_2^2$, then

$$\frac{\partial f}{\partial x_1} = 2ax_1 + bx_2 \text{ and } \frac{\partial f}{\partial x_2} = bx_1 + 2cx_2.$$

Total Differential

If all the x's are varied by a small amount, the total effect on y will be the sum of effects. This is defined as follows:

$$dy = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + ... + \frac{\partial f}{\partial x_n}dx_n.$$

Implicit Functions

The implicit function $f(x, y) = 0$ has a total differential of

$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 0 \Leftrightarrow f_x dx + f_y dy = 0.$$

Thus, we have

$$\frac{dy}{dx} = -\frac{f_x}{f_y}, \text{ where } f_y \neq 0.$$

Examples Using Microeconomics Concepts:

i) The market demand function is $q = D(p)$. The inverse form of the demand function is $p = D^{-1}(q) = p(q)$. Given this, we can write the monopolist's total revenue function as:

$$TR(q) = pq = [p(q)]q$$

Using the product rule (Rule 7), we can obtain marginal revenue as:

$$MR(q) = \frac{dTR(q)}{dq} = \frac{dp}{dq}q + p(q)\frac{dq}{dq}$$

or

$$MR(q) = p^{'}(q)q + p$$

ii) Let q = 100 - p and the total cost function be $TC(q) = 25q$. Note that $TR(q) = pq = 100q - q^2$. Thus, we can define the total-profit function as:

$$\pi(q) = TR(q) - TC(q) = 100q - q^2 - 25q = 75q - q^2$$

Recall that profit-maximizing p and q are obtained at the point of $MR(q) = MC(q)$. Hence, we have:

$$TR^{'}(q^*) = TC^{'}(q^*) \Leftrightarrow 100 - 2q^* = 25$$

$$q^* = 37.50 \text{ and } p^* = 100 - 37.50 = \$62.50$$